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ABSTRACT

Eighteen research reports related to mathematics education are abstracted and analyzed. Four of the reports deal with aspects of learning theory, five with topics in mathematics instruction (history of mathematics, exponents, probability, calculus, and calculators), four with teacher characteristics, and one each with testing, student interests, goals for teaching mathematics, and classroom practices. Research related to mathematics education which was reported in RIE and CIJE between July and September 1977 is listed. (MN)

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RELATIONSHIPS BETWEEN VARIABLES IN LEARNING AND MODES OF PRESENTING MATHEMATICS CONCEPTS. Abkemeier, M. K.; Bell, F. H. International Journal of Mathematical Education in Science and Technology, v7 n3, pp257-270, August 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Larry Sowder, University of Northern Illinois, Dekalb, Illinois.

1. Purposes

(a) To determine whether figural or symbolic modes in programmed materials on functions give superior performance on immediate learning or on one-week retention.

(b) To determine whether there are interactions between the figural and symbolic modes and either sex or mode-preference.

(c) To explore relationships between these modes and several of Guilford's structure-of-intellect (SI) aptitudes.

2. Rationale

The attractiveness of the SI model to researchers in aptitude-treatment interaction (ATI) studies has led to several studies which have not given uniform results. Some have used only a limited number of aptitudes. None of the studies cited considered the learner's "preferred" mode of instruction.

3. Research Design and Procedure

One hundred ninety-nine beginning algebra students were screened for suitable "abstraction age" (Shipley abstraction test) and prior knowledge of the mathematical concept of function. The 160 survivors were given a mode-preference test constructed by the authors and then were randomly assigned to a 3-class-session use of programmed materials involving either a figural mode (arrow diagrams, function machines) or a symbolic mode (symbols, formulas, sets of ordered pairs). A 100-point learning test constructed by the authors was given at the next class and a second 100-point test, a retention test, was given a week later. (Each test presented some items in figural, symbolic, and "neutral" modes.) During the week between the learning test and the retention test, 11 SI-inspired aptitude tests were given, 5 for figural aptitudes, 5 for symbolic aptitudes, and 1 for a semantic aptitude.

4. Findings

(a) Two-tailed t-tests ($p = 0.05$) showed no differences between the learning test or retention test performances of the 87 figural Ss or the 73 symbolic Ss. The 43 figural-mode males, however, performed better than the 39 symbolic-mode males on the learning test items presented in a figural or a "neutral" mode.

(b) No interactions between mode and either sex or preferred mode were found in separate two-way analyses of variance ($p = 0.05$). The 78 females did perform statistically better than the 82 males (e.g., learning test: females--55.9%, males--50.0%; retention test: females--67.8%, males--59.2%).

(c) Various multiple and simple linear regression analyses suggested that the aptitude, divergent production of figural systems (DFS; measured by the Making Objects test), was the best single predictor, giving a positive coefficient for the figural group and a negative coefficient for the symbolic group and accounting for the major part of the variance (ca. 50% for the retention test scores, even after the other 10 measures had been entered). Cognition of symbolic systems (CCS; measured by the Letter Series test) was the best single predictor within the symbolic group. Cognition of semantic systems (CMS; Necessary Facts test) displayed some strength for the figural group.

5. Interpretations

(a) "In general, findings indicate (although not conclusively) that for students of approximately 15 years of age, especially males, a figural instructional mode is preferable to a symbolic mode" (p. 268).

(b) "Although it appears that different verbal aptitudes are needed to learn from figural and symbolic instructional materials, the authors hesitate to draw definitive conclusions based upon the results of this study" (p. 269).

(c) It is possible to some extent to "design instructional programs to suit the learner's mental aptitudes" (p. 269), as the DFS results suggest. However, not all the predictors behave as they should.

(d) DFS should be considered as a predictor of achievement with similar materials.

(e) Experimenter-designed aptitude measures seem to be required; difficulties of the usual "parallel" measures of aptitudes were quite different in this study.

(f) Future ATI studies should consider the information processing mode of the subject.

Critical Commentary

(a) It is a relief to an abstractor to find an article that is clearly written and reported in such detail that a reader has a very good idea of what the study involved. Thank you, authors. Nonetheless, an excerpt of the instructional materials and sample test items would have added even more to the report's understandability. Post-test measures of 50-60% make one wish to see the items. The length of the article probably resulted in the omission of the correlation coefficients of the measures, something of interest to ATI and SI devotees.

(b) Some other randomization procedure could surely have resulted in a better split than the 87-73 one obtained in the study.

(c) The authors were honest in admitting that unfortunately the aptitude tests were given after the treatments (due to school constraints). From a strict design viewpoint, this disaster gives defensible grounds to anyone who chooses to reject the study's assertions about using the measures as predictors.

(d) Authors do not always write as though they understand the statistical analyses they are using; these authors did. Especially noteworthy is the attention they gave to the assumptions for statistical tests (not all of which were met). On the other hand, they seemed to ignore the questionable practice of isolating single variables in a context rich in other, possibly confounding, information. After all, quite different hyper-planes might give similar projections onto a single dependent variable-aptitude measure plane. The authors can hardly be faulted for (apparently) giving attention only to models linear in the variables since this practice is so common; yet, one would hope to find at least minor explorations of other models.

One last point on the analyses: rather than the separate 2-way analyses of mode vs sex and mode vs preferred mode, would not a 3-way analysis--perhaps even multivariate--of mode vs sex vs preferred mode have been more instructive? And, why were there only "figural" and "symbolic" categories of preferred mode? Would a "no-preference" category as a third level have made the definition of the other two categories questionable?

(e) Whither ATI research? Let us re-hash some of the concerns provoked by ATI results. Can treatments be designed which do involve different levels of aptitudes strongly enough so as to be differentially effective? Are the aptitude measures sensitive enough? Perhaps we should do teach-test versions of determining aptitudes instead. This study says, "Yes, perhaps." Some investigators refuse to admit that aptitudes are immutable. Hence, even if we could design such treatments and measure aptitudes, should we play to a student's strength and allow important but weaker areas of cognitive functioning to atrophy or remain undeveloped?

A STUDY OF PUPILS' PROOF-EXPLANATIONS IN MATHEMATICAL SITUATIONS. Bell, A.W. Educational Studies in Mathematics, v7 n1/2, pp23-40, July 1976.*

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Jeremy Kilpatrick, University of Georgia.

1. Purpose

The study was undertaken "to analyze pupils' attempts to construct proofs and explanations in simple mathematical situations, to observe in what ways they differ from the mature mathematician's use of proof, and thus to derive guidance about how best to foster pupils' development in this area." (p. 23)

2. Rationale

The widest variation across countries in mathematics teaching is probably in their approaches to proof. This variation is based on a tension between recognizing that deduction plays an essential role in mathematics and acknowledging that usually only the most capable pupils understand deduction. Bell argues that proof grows out of the (gradually externalized) testing that accompanies the development of generalizations. An awareness of the public status of knowledge and the value of public verification is necessary if pupils are to appreciate the purpose of formal proof, and cooperative research activity by the class is probably the best mechanism to develop such an awareness. Knowing characteristics of pupils' proof-explanations may be helpful to instruction.

In an earlier work Bell identified stages in pupils' understanding of deductive proof. In the present study he used a greater variety of problems and made a deeper analysis of responses.

3. Research Design and Procedure

Ten numerical and geometrical problems requiring the explanation and justification of a generalization were given to 30-40 fourth-form pupils (aged 14-8 to 15-8) of all levels of ability selected from one grammar and two comprehensive schools. The article deals primarily with pupils' responses to two problems: One and the Next (in which the pupil must explain, for numbers up to 15, why just one of two consecutive numbers and their sum is a multiple of three) and Triangles (in which the pupil must generate the complete set of equilateral triangles whose vertices lie on a given equilateral triangle), with additional

* This report is apparently based on Chapter 9 of Bell's doctoral thesis, "The Learning of General Mathematical Strategies" (University of Nottingham, 1976).

illustrations from Add and Take (in which the pupil must explain why, if an arbitrary number between 1 and 10 is added to and subtracted from 10, the sum of the resulting two numbers is always the same). The procedure for obtaining "scripts" from the pupils is not explained in the article, but the doctoral dissertation indicates that the problems were put into pairs in half-hour tests, with pupils writing out their responses.

Each script was assigned to a category depending on the pupil's predominant approach to the problem. The six "empirical" categories ranged from the failure to generate correct examples or to comply with the given conditions through the checking of a full, finite set of cases. The seven "deductive" categories ranged from the failure to use correct examples to test the general statement through a complete and connected argument from data and accepted facts or principles to the conclusion. Thirty-two scripts for each of the two primary problems were also categorized by a second person; only two scripts required reclassification by mutual agreement. Then scripts originally classified at the bottom of the "empirical" hierarchy because of failure to comply with the problem's conditions were re-categorized, if possible, by accepting the pupil's interpretation of the problem and then classifying the approach.

4. Findings

The percentage of the 32 pupils whose script fell into each category is given for the two main problems. An appendix contains three sample scripts for Triangles, two sample scripts for One and the Next, and one for Add and Take. (Excerpts from these and other scripts were used to illustrate the meaning of the various categories.)

One and the Next was a hard problem for the pupils: 48% (actually 50%) failed to interpret the problem correctly, 9% asserted the generalization on the basis of a few cases, 19% correctly checked all 14 cases, and the remaining 31% made some incomplete attempt at a deduction. (The percents add to more than 100; an unexplained inconsistency between a sample size of 35 in the dissertation and 32 in the article appears to account for the excess.) Triangles was also a hard problem: 9% mistook "equilateral" for "congruent", 28% generated triangles whose vertices were not on the given triangle, and only one pupil's response reached the level of incomplete explanation.

5. Interpretations

Bell argues that many of the pupils' failures arose from their inability to coordinate all of the data in the problem. The complexity of the problems appeared to overload their information-processing capacity. Bell suggests that experience with problems of the sort used in the study together with Polya's hints for understanding a problem may be a means of countering the effects of problem complexity. He also mentions a need to make pupils more aware of the usefulness of algebraic approaches in

number situations, of a systematic ordering of examples, and of repeated attempts to take fresh points of view. Conscious attention to strategies and discussion of the nature of proof-explanations are hypothesized to be effective in improving the abilities to generalize and prove.

Critical Commentary

Bell is to be commended for his penetrating analysis of some of the difficulties pupils have in formulating and verifying mathematical generalizations. He has made a good stab at erecting an analytic framework, but the category scheme appears to be ad hoc. Differences between problems and between the dissertation and the article in the categories used suggest instability in the scheme; it should be tried with fresh sets of problems and pupils.

The argument for problem complexity as a critical factor in failure to provide adequate proof-explanations would have been strengthened by evidence that pupils could offer such explanations when problems were not so complex. Although the dissertation does not present such evidence, it does provide a fuller, and a more convincing, analysis than the article does.

THE INTERPRETATIONS OF STATEMENTS IN STANDARD LOGICAL FORM BY PRESERVICE ELEMENTARY TEACHERS. Damarin, S. K. Journal for Research in Mathematics Education, v8 n2, pp123-131, March 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Lars C. Jansson, University of Manitoba.

1. Purpose

The major purpose of the study was to determine the extent to which preservice elementary teachers use mathematical logic in their interpretation of mathematical statements in standard logical form. A secondary purpose was to investigate which changes in the truth values of component simple statements would be made by subjects when asked to alter the situations so as to change the truth value of a compound statement.

2. Rationale

The importance of logical reasoning, not only as a long-term goal, but as a tool for learning and doing elementary mathematics, appears to be increasing with the current emphasis on problem solving. Earlier studies established that preservice elementary teachers experience difficulty in handling logical connectives. The present study relates its findings to earlier studies in a number of ways. Among these ways are (1) exploring a new type of logical task, and (2) restricting itself to content in the domain of mathematics. The latter was done in a way so as to be independent of prior mathematical knowledge. Within this content domain several hypotheses of previous researchers regarding similarities in the handling of conjunction, conditional, and biconditional are examined.

3. Research Design and Procedure

A 32-item "Truth Value Inventory" (TVI) was administered to 70 preservice elementary teachers. A three-factor ANOVA was used to analyze connectives (four levels: conjunction, disjunction, conditional, biconditional), truth values (four levels: TT, TF, FT, FF), and subjects (70 replications). A sample item pair is given in Figure 1. In this instrument, each item consists of two parts. If truth value is considered as a function, then standard inference tasks are of the form, where C is any logical connective: TV(pCq) = given, and, say TV(q) = given, TV(p) = ? Using this symbolism, the first part of each item may be phrased:

TV(p) = given
TV(q) = given
TV(pCq) = ?

and the second part: TV(pCq) = given
select: TV(p), TV(q).

Responses to true-false parts of items were scored as: true = 3, omit = 2, false = 1. The four connectives and four truth-value levels give a 4x4 matrix of items and each type was duplicated to make up the instrument. Pairwise comparison of means was carried out by Tukey's hsd method.

The four statements on this page refer to these sets:

Set A Set B

Read each statement carefully; then circle its truth value (T if true, F if false). Then circle the objects you would move to the other set in order to change the truth value of the statement. Remember to move as few as you can and still change the truth value!

1. If is in set A then is in set B. T F

Move

2. is in set A and is in set B. T F

Move:

Figure 1. Abbreviated directions and sample items from Truth Value Inventory. From Journal for Research in Mathematics Education, March 1977, p. 125.

Responses to the second parts of items, or "moves" were scored according to the system: 0 = no move made, 1 = move only the object in the first simple statement; 2 = move only the object in the second simple statement; 3 = move both objects mentioned in the compound statement; 4 = any deviation from the above. These scores were combined with true-false scores to yield a 15-point nominal scale. Frequency distributions were computed for each item. Examination of these distributions suggested post hoc hypotheses that were tested by

the chi-square statistic. Presumably the ANOVA is calculated on the basis of the first (true-false) scores only.

4. Findings

Sixteen tetrachoric correlations were computed and found to be acceptable as a measure of item-pair reliability. The ANOVA indicated that (1) the proportion of compound statements declared to be true varies as a function of the connective, and (2) the proportion of compound statements declared to be truth values as a function of the combination of truth values of the constituent parts. When comparisons are made among connectives with truth values held constant, there is only one case (false-false) for which the conjunction differs from the biconditional, and in no case does the conditional differ from either the conjunctive or the biconditional. The disjunction differs from all other forms in the true-false and false-true cases, and from the biconditional when both simple statements are false.

Similarly, when comparisons are made among input truth values with connectives held constant, there is no significant difference between true-false and false-true statements. For conjunctions there is no difference between false-false and false-true or true-false; for implications false-true and false-false are not significantly different.

With respect to the second part of each item, the following hypotheses were retained:

(1) The "move" part of an item will be omitted more often when the statement is declared true than when it is declared false.

(2) In balanced statements (conjunctions, disjunctions, and biconditionals with both parts true or both parts false), the move made will involve the second simple statement more often than the first.

Of the omissions, 63% were in true-true situations. Omissions were distributed approximately equally over connectives.

5. Interpretations

Confirming earlier studies, a major finding of this investigation was that the subject tended to treat conjunctive, conditional, and biconditional statements in the same way, declaring them to be true only if both parts were true. Roughly half of the population also treated disjunction in this manner.

The pattern of omissions in the move portion of the item may be either a hesitancy to select from among several responses or it could

result from subjects' uncertainty as to whether the falsity of one simple statement suffices to make the compound statement false. This calls for further investigation. It is hypothesized that (1) preservice teachers seem to be more confident when adjusting situations to make false statements true than when making true statements false, and (2) the second statement in a compound sentence seems to have a greater effect on subject behavior than the first.

Critical Commentary

(1) The sample items presented appear to this reviewer, in their two parts, to be valid exemplars of the two types of logical tasks under study. However, no information on test validity is provided and one could debate the validity of the second part of the item. The request to move an object from one set to another is not the same as selecting TV(p) and TV(q). The tasks are more complex than this. Nevertheless, this is an excellent try at a different problem.

(2) The report of the study is unclear as to how the instrument scoring system (see #3 above) is used in relation to the hypotheses and the analyses. The former deal with proportions and the latter with a continuous score. Although there are equal numbers of items in each T-F category, item scoring in relation to these is not stated. Similarly, the use of the 15-point nominal scale is not explicated. Replication would be difficult at best without more information.

(3) The effort here to deal with logical operators in a subject matter context is to be commended, although some may question the mathematical nature of the test items. The standard inference items might also be tested within mathematical content.

The problem of content testing is a difficult one. It is, I think, generally agreed that logical processes do not operate in a vacuum, that is, they operate on particular content. The difficulty is thus in dealing with content that is independent of prior student experience. There is real payoff here for curriculum development in knowing how subjects interpret logical operators within the domain of mathematics.

BEGLE REVISITED: TEACHER KNOWLEDGE AND STUDENT ACHIEVEMENT IN ALGEBRA. Eisenberg, Theodore A. Journal for Research in Mathematics Education, v8 n3, pp216-222, May 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Mary Grace Kantowski, University of Florida.

1. Purpose

This study is a replication of the Begle (1972) study to determine if a significant positive correlation exists between teacher knowledge and student performance in Algebra I.

2. Rationale

The author notes a selection bias in the Begle study: 492 teachers began the study, but the sample was reduced to 308 by the conclusion of the study. Moreover, the author notes the motivation factor involved since all teachers who took part in the Begle investigation were volunteers from among participants in National Science Foundation Institutes.

Since "teacher training programs are built on the assumption that teachers do influence student learning," the author hypothesized that the selection bias in the Begle study could have affected the results. Thus, the decision was made to replicate the study with an "unbiased" sample.

3. Research Design and Procedure

Of the 52 teachers in junior high schools in Columbus, Ohio offering Algebra I, 42 teachers whose schools were willing to participate were asked to take part in the study. Twenty-eight teachers participated. Twenty-five finished the study.

Each teacher took the Algebra Inventory Form B examination (Begle, 1972). Two author-constructed logic tests and the Mathematics Inventory I Examination (Begle, 1972) were administered to Algebra I students of these teachers in the fall. During the winter other data (e.g., grades, aptitude measures) were gathered from student files. The Mathematics Inventory III and IV (Begle, 1972) were administered to the students in the spring. Regression equations were then used to predict expected Mathematics Inventory III and Mathematics Inventory IV scores for the students of each teacher. Effects of teacher with respect to algebraic concepts and algebraic skills and correlations between teacher effects and teacher variables were computed. Student scores of "Begle teachers" and of "Columbus teachers" were compared for Mathematics Inventories I, III, and IV.

4. Findings

The "best" eight variable models for predicting algebraic skills and algebraic concepts were listed. Among the best predictors of algebraic skills were grade earned in the last English course and several measures of

academic aptitude using the Short Form Test of Academic Aptitude (SFTAA) and the California Comprehensive Test of Basic Skills (CCTBS) scales. Among the best predictors of Algebraic concepts were Begle's Mathematics Inventory I, grade earned in last mathematics course, the SFTAA non-language scale and three CCTBC mathematics scales.

There were no significant correlations between the teacher variables and effect of teacher on algebraic skills or algebraic concepts on the part of the students.

Critical Commentary

(1) The author seems to have encountered difficulties similar to those faced by Begle in the sample selection. Of the 52 Algebra I teachers, the final results used only 25--fewer than half. The question arises whether the author was, in fact, able to accomplish what he intended to do with similar sampling difficulties. In fact, the problem was more pronounced with a smaller sample.

(2) The purpose of the author-constructed test was not clear.

(3) It is not clear why, in comparing student scores of "Begle teachers" to "Columbus teachers," only 13 teachers were selected for comparison using Inventory III and only 10 selected for comparison using Inventory IV, since the entire sample of Columbus students was used for comparing student scores on Inventory I. Moreover, the scores of the three teachers who dropped out of the study were used in the comparison using Inventory I. Why?

(4) In looking at the "best predictors" for success in algebraic skills, only one mathematics-related variable (CCTBS arithmetic concepts) appeared in the eight-variable model. However, among the "best predictors" for algebraic concepts, five of the eight variables in the eight-variable model--including Mathematics Inventory I and "grade earned in last mathematics class"--were good predictors. This observation raises interesting questions overlooked by the author.

Reference

Begle, E. G. Teacher Knowledge and Student Achievement in Algebra. SMSG Reports, No. 9. Stanford: School Mathematics Study Group, 1972.

THE ACHIEVEMENT OF EDUCABLE MENTALLY RETARDED ON THE KEY MATH DIAGNOSTIC ARITHMETIC TEST. Goodstein, H. A.; And Others. Journal of Special Education, v10 n1, pp61-70, Spring 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Carol A. Thornton, Illinois State University.

1. Purpose

To determine, for an EMR population:

(a) The accuracy of the scaling of Key Math subtest items.

(b) The relationship of item performance to subtest and total test performance.

(c) Whether subtest scores represent unique dimensions of total test performance.

(d) Whether a relatively flat profile of performance over the 14 subtests is obtained.

(e) Whether children at succeedingly older ages perform at approximately grade equivalent expectancies (based on M.A. rather than C.A.).

2. Rationale

Although the pool of items originally generated for Key Math was drawn from studies of educable mentally retarded children, final test items were normed on populations of regular class students. Jones (1973) points out the limitations of using norm-referenced tests with exceptional populations when the norms are not specifically developed for those populations. Since Key Math is used in the assessment of EMR children, it is necessary and useful to generate additional data on the performance of an EMR population on Key Math.

3. Research Design and Procedure

The 227 subjects for this study were randomly selected from self-contained EMR classrooms in two geographically different school districts. The sample was drawn from a population in which the mean IQ was 65.8 (S.D. = 8.9); the mean chronological age was 11 years 4 months (S.D. = 5 years 1 month); and the mean mental age was 7 years 6 months (S.D. = 4 years 0 months). By pooling children whose grade expectancies (based on M.A.) fell within a range defined by grade level $\pm$ 0.5, the sample was partitioned in the following manner:

Subgroup	N	Grade Level Expectancy
a	40	0.0 (M.A. < 5 years 6 months)
b	43	between 0.5 and 1.5
c	50	between 1.5 and 2.5
d	38	between 2.5 and 3.5
e	30	between 3.5 and 4.5
f	19	between 4.5 and 5.5

The Key Math Diagnostic Arithmetic Test was individually administered to each subject by trained graduate students and para-professionals. The Key Math manual directions were strictly followed, so testing on each of the 14 subtest areas was terminated when three items in succession were missed by a subject.

The accuracy of the present scaling of items within subtests for the EMR population was ascertained by calculating the percentage of the 227 children passing each item and subsequently examining those distributions of items which were at least 5 percentage points more difficult than the next item on the subtest. Biserial correlation analysis was used to assess the relationship of item performance to subtest and total test performance. To determine whether subtest scores represented unique dimensions of total test performance, subtest scores of the 227 children were then factor-analyzed using principle components--the varimax rotation technique. The mean performances of each subgroup on each of the 14 subtests were also calculated and the profile was plotted against the grade level expectancy for each subgroup. Profile trends rather than formal statistical tests were used to determine whether EMR children at succeedingly older ages performed at approximate grade equivalent expectancies (based on M.A. rather than C.A.).

4. Findings

The analysis of subtest items revealed that, for the EMR population sampled, items were quite accurately scaled in order of increasing difficulty. Only 13 out of 209 items were "out of sequence." The Geometry and Symbols and Time subtests had item difficulty sequences most at variance with published norms.

Subtest	Item Numbers		
Geometry and Symbols	4	——more difficult than—→	6
	7		9
	11		12
	14		15
Numeration	7		9
Numerical Reasoning	4		5
Word Problems	6		7
Measurement	6		7
	12		13
Time	3		4
	6		7 and 8
	13 and 14		15

This analysis also revealed that, in terms of item difficulty for the sample, wide gaps often appeared between items or clusters of items within subtests. On Numeration 58% of the children passed item 13, but only 21% passed item 14. On Fractions 60% passed item 2, but only 14% passed item 3. Geometry and Symbols had a gap of 28 percentage points between items 11 and 13. Division had a gap of 47 percentage points between items 2 and 3, and Money had a 39 percentage point gap between items 4 and 5.

Biserial correlational analysis indicated that, with three exceptions, all items that did not fall into marginal areas of difficulty contributed positively to the composition of the individual subtest scores and the total test score. Significant factorial validity was found for the claim of uniqueness of the 14 Key Math subtests (each being identified by a different subtest loading in excess of $\pm$ .60). However, only limited validity could be found for the grouping of subtests by areas.

Subtest profiles revealed a relatively flat pattern of slight underachievement across the 14 subtests, with two exceptions. Performance on the Missing Elements subtest was markedly <u>below</u> that on other subtests for the subgroups (with the exception of subgroup e, where performance was still below expectancy and among the poorest). The Money subtest performance was consistently <u>above</u> grade level expectancy (with the exception of subgroup f, where a score of 4.8 was achieved). For all subgroups, performance on the Money subtest was higher than that on any other subtest. The one outstanding trend revealed by the graphs of the six subgroups across the 14 subtests is the incremental deficit in performance (relative to grade-level expectancy) of children at succeedingly older ages.

5. Interpretations

(a) Since Keymath subtest items were not randomly administered and not all items were given to all children, the data generated cannot be interpreted as a "renorming" of Key Math.

(b) Discrepancies in item difficulty sequencing suggest that an examiner of EMR children may want to proceed to the next item when a ceiling has been established for a child on an item which, across a representative population, has been demonstrated to be more difficult than the succeeding item.

(c) While the difficulty level of the basal items on most subtests is acceptable for EMR children (exceptions being Multiplication and Missing Elements), the children tend to reach the ceiling more rapidly. A lack of items in the mid-range of the tests prevents the fullest use of the data to provide diagnostic information that will effect programming decisions for EMR children. For this same reason the instrument tends to lose power to discriminate achievement change (which proceeds at a slower rate) for EMR children.

(d) Profile deviations above grade-level expectancy performance for many EMR children on the Money subtest seems to be directly related to the emphasis placed on teaching money in EMR classrooms. The marked underachievement of the children on Missing Elements can be traced directly to the lack of adequate basal items.

(e) The pattern of slowed growth by succeedingly older subgroups of children in this population (represented in the limited gain both in grade level equivalents of the raw scores of the children on most subtests after expected grade level 3.0 and the limited number of items mastered by 50% of the subgroup children after that expected grade level) follows the pattern detected by Burrow (1969) in a study of reading and listening comprehension among EMR children at succeedingly older age levels.

Critical Commentary

This status study was well designed and executed to address a matter of practical significance to special educators. The difficulties pinpointed by the researchers in the use of Key Math with EMR groups are characteristic of any norm-referenced instrument developed and normed on a population of children whose rate and range of achievement exceeds that of exceptional learners.

Two areas of further exploration are suggested to the abstractor by the present study:

(1) Developmental work in constructing a more viable diagnostic

instrument for exceptional subpopulations might well begin by designing items of appropriate difficulty to fill the gaps between items and item clusters identified on subscales by Goodstein and his colleagues.

(2) Further examination of age trends vs. grade level expectancy in profile analyses with larger, carefully selected samples may be a fruitful area of inquiry. It is not clear to what extent the incremental deficit in performance by older groups of children should be attributed to mental deficiencies, to the effects of teacher expectancies, to curricular offerings and instructional practices, or to the role of early failure experiences.

Within the limits set out by the researchers, the present study itself was clearly reported. Only a slight ambiguity was noted in the overall N of 227 when tallied against the total of 220 in the composite subgroup.

References

Burrow, W. Listening and Reading Comprehension in Special Class Educable Mentally Handicapped Children at Selected Developmental Levels. Unpublished Ph.D. thesis, University of Connecticut, 1969.

Jones, R.L. Accountability in Special Education; Some Problems. Exceptional Children, v39, pp631-642, 1973.

STUDENTS' INTERESTS IN PARTICULAR MATHEMATICS TOPICS. Hogan, Thomas P. Journal for Research in Mathematics Education, v8 n2, pp115-122, March 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James M. Sherrill, University of British Columbia.

1. Purpose

". . .to identify the degree of self-expressed student interest in a great variety of particular mathematics topics and to determine changes in these interests across grade levels."

2. Rationale

Measurement of academic interests in the past has tended to concentrate on global statements about school subjects. Such measurement yielded little information about specific likes and dislikes within a subject area--information that is critical for understanding children and for suggesting more fruitful research.

Aiken, in his review of research on attitudes toward mathematics, concluded "...that the concept of a general attitude toward mathematics should be supplemented with that of attitudes toward more specific aspects of mathematics...".

3. Research Design and Procedure

The Survey of School Attitudes (SSA) was administered to 13,401 students in 12 school systems in 10 states. Each of the two levels (Primary: grades 1-4 and Intermediate: grades 4-8) and two forms (A and B) of the SSA had 18 mathematics items. Each item consisted of a picture and a verbal statement about the picture. For example, the item might have shown a set of geometric figures, and the student responded to the question "Do you like to work with figures?" The students marked each item on a three-point scale: like, indifferent, dislike. The items were then distributed according to the percentages of students marking the "like" response for each item.

Each of the primary items was compared according to its ranking by grades 1-2 and grades 3-4. An analogous comparison was made of the ranking of the intermediate items between grades 5-6 and grades 7-8. The items were then distributed according to the change in the percentages of students marking the "like" responses.

4. Findings

In the lower grades, the average percentage-liking figure for all items combined was 58%. However, for individual items, considerable diversity was noted; the percentage-liking figures ranged from approximately 40% to 80%. The most favored topics included most of the measurement items, several basic numeration topics, and story problems. Among the least favored topics, the geometry items stand out. Items on sets and number sentences also tended to be low.

At the upper grades, the average percentage-liking data for all items combined was slightly under 40%. As with the primary level, the variation on the items was considerable, ranging from about 26% to 60%.

The most favored items for the upper grades were a variety of computation items. It is hard to categorize the least-favored items; no one group of items was ranked low relative to the group of students. However, of the 36 items on the two forms for the intermediate group, only 4 items had a percentage-liking figure over 50%.

In the lower grades, there was an increase in liking for doing addition problems and virtually no decrease in liking for doing subtraction problems. Many of the favored items in numeration tended to remain fairly stable. On the other hand, leading the decline in interest are two types of items that were among the least favored on the whole: geometry and sets. Some, but not all, of the measurement items showed appreciable declines.

In the upper grades, working with graphs and some computation items showed little loss or even slight gains in degree of student interest. Leading the decline in interest were items in geometry, word problems, and a variety of numeration items.

5. Interpretations

The average percentage-liking figure for all items in the lower grades, (58%) indicates "a generally favorable attitude toward mathematics topics."

At the upper grades, the average percentage-liking figure for all items combined was slightly under 40%. The author points out, however, that "It should be recalled that students responded on a three-point scale: like, indifferent, or dislike. Thus, the fact that 40% of the students reported liking a topic does not mean that 60% disliked it. In fact, the average response to the mathematics topics at the intermediate level was on the positive side, although the responses are clearly less favorable than at the primary level."

The data on changes in interest in particular mathematics topics confirm the notion that as students move through school, they develop less favorable attitudes toward mathematics. However, data presented

in Table 4 (p. 119) indicate that (a) this is not universally true and (b) there is considerable diversity in the degree of change with respect to different mathematical topics.

The author also stated that "An interesting hypothesis suggested by the data is that the harder the content of an item is perceived to be, the less students will like it."

From a practical viewpoint, two implications should be apparent. First, teachers should be made aware of the differential interest values of various mathematical topics. Second, the differential patterns of change in liking for certain topics across grade levels suggest that program evaluations that incorporate assessment of attitudinal variables may be improved by analyzing changes in attitude toward specific mathematics topics.

Two major research questions were generated:

(1) What factors might account for, or at least be associated with, the differences in attitude toward particular topics?

(2) What factors cause or are associated with the differential changes in attitude toward specific topics?

Critical Commentary

First, a technical flaw in the article: item 66 is not ranked in Table 2, form A, primary.

Second, a very general comment: I am very pleased to see an article of this type appear in the JRME. The article had no statistical analyses. The data are simply presented, statements are made about the data, and suggested research is listed. The article is one to generate items of interest for further research. It is a worthwhile article for the direction it gives to future research.

Of course, without the statistical analyses for support, one must be very careful about the interpretations one makes of the data. There are three interpretations that need to be discussed.

(1) While the author said that an average liking figure of 58% indicated "a generally favorable attitude toward mathematics topics", he concluded that an average liking figure of 40% is not really unfavorable. He suggested that since there were three categories (like, indifferent, dislike) it was possible (though no data were given) that very few marked "disliked" and it was certainly true that 60% did not mark "dislike". He goes on to say, "In fact, the average response...was on the positive side..." If "indifferent" is interpreted to be neither positive nor negative, then

the results could only be interpreted as negative. The only way I see that he could make such a statement is that he interprets "indifferent" as being half positive and half negative.

(2) He points out that there does not seem to be a pattern for the least-favored items at the intermediate level. He cited as an example that one geometry item was quite low (B18 - 30%) while another was relatively high (A18 - 48%). The reader should be cautioned about the importance of the word "relatively." On a form A only one item is above 50%; i.e., none of the items "do very well".

(3) Hogan stated that "An interesting hypothesis suggested by the data is that the harder the content of an item is perceived to be, the less students will like it." The key word in the quote is "perceived" and even the word must be interpreted very carefully. The students did not work addition problems; rather, the students were asked questions such as "Do you like to work hard addition problems?" Whether the students were reacting to the "content of the item" or to the description that the addition is "hard" is very difficult to determine.

Hogan does a very good job in reporting a study that had a carefully constructed sample and, although no statistical analyses are offered, the data direct us to more involved studies concerning the "whys" of the patterns that exist in the data.

THE EFFECTS OF HISTORY OF MATHEMATICS ON ATTITUDES TOWARD MATHEMATICS OF COLLEGE ALGEBRA STUDENTS. McBride, Cecil C.; Rollins, James H. Journal for Research in Mathematics Education, v8 n1, pp57-61, January 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James H. Vance, University of Victoria, Victoria, B.C.

1. Purpose

To study the effectiveness of incorporating items from the history of mathematics into classroom discussions of mathematical topics as a technique for improving attitudes toward mathematics of college algebra students.

2. Rationale

Research indicates a steady decline of student attitudes toward mathematics through the high school years (Begle, 1973). Thus the instructor of first-year college mathematics has a particularly challenging task in promoting positive attitudes toward the subject.

Although there has been considerable work done in the area of attitudes toward mathematics (Aiken, 1970), relatively little substantive research investigating methods of improving student attitudes has been reported. One suggested technique for improving attitudes is that of incorporating items from the history of mathematics into classroom discussion of mathematical topics. While the idea has been endorsed by the National Council of Teachers of Mathematics and classroom experience with the method has been favorable (Jones, 1969), there is a lack of empirical evidence relating to its effectiveness.

3. Research Design and Procedure

The subjects were students who enrolled in four sections of a college algebra course at Louisiana Tech University for the 1972-73 winter quarter. Two sections were designated as the test group and two as the control group. Two instructors, selected on the basis of similarities in age and teaching experience, each taught one test class and one control class. During registration, the subjects were randomly assigned to groups. Assignment of teachers to classes and the designation of type of class were determined by coin toss. Initially 30 students were enrolled in each class, but there were only 67 who completed the course and for whom both pretest and posttest attitude scores were obtained (Table 1).

Table 1

Class and Group Sizes

	Teacher A	Teacher B	Total
Test Group	21	14	35
Control Group	17	15	32
Total	38	29	67

Twenty-five items from the history of mathematics, each taking about five minutes of class time, were used in the test classes with associated topics in the course. The course consisted of 75-minute classes meeting three times per week for about 12 weeks. Approximately one item was used per class meeting. A textbook devoid of historical content was used in all classes to guard against contamination.

The Revised Math Attitude Scale (Aiken, 1972), a Likert-type scale with a range of 80 points, was administered to all subjects at the beginning and at the end of the 12-week quarter. A student's attitude change was taken as the difference between pretest and posttest scores.

The study involved a 2x2 treatment-by-teacher factorial experimental design. Analysis of covariance, with initial attitude scores as the covariate, was used to test for differences in attitude change due to treatment and teacher and for treatment-teacher interaction.

4. Findings

The mean attitude change for each class and each group is shown in Table 2.

Table 2

Mean Attitude Change for Classes and Groups

	Teacher A	Teacher B	Total
Test Group	2.7143	-1.5714	1.0000
Control Group	-1.6471	-.75333	-4.4063
Total	0.7632	-.6552	

The analyses of covariance revealed a significant difference in

attitude change due to treatment ($p < .0156$) and also to teacher ($p < .0162$). Neither analysis indicated a significant treatment-teacher interaction ($p < .7216$).

5. Interpretations

The primary conclusion of the study was that the technique of using items from the history of mathematics was effective as a means of promoting positive attitudes toward mathematics for college algebra students.

The investigators suggested two ways such use of historical materials might contribute toward the development of favorable attitudes: (1) the injection of human interest in the subject, through mention of activities and beliefs of famous mathematicians containing traces of humor and irony; (2) student perception of the use of the material as signifying interest or enthusiasm on the part of the teacher. It was reported that both teachers felt that the time involved in using the items was well spent.

Replication of the study at the high school and general mathematics college levels was recommended.

Critical Commentary

Attitude studies such as this which investigate and seek to identify classroom techniques for improving student attitudes toward mathematics are to be commended. Perhaps the most valuable outcome of this study is the set of historical items, which, it would appear, can be used by instructors of similar courses without great expense or preparation time to make their subject more interesting to teach as well as to learn. The fact that the difference in attitude change between the test and control groups was statistically significant is perhaps a secondary reason for recommending use of these or similar materials.

The statistical design confused this reader. The hypotheses and results (Table 2) are presented in terms of attitude change, while the analysis of covariance tested the differences between final, adjusted group scores--not change per se. A reference for this design would be helpful. In any case, it is important to note (Table 2) that the difference between groups resulted more from a decline in attitude scores of the control group than from an improvement in attitude scores of the test group.

As the investigators pointed out, the high percentage of students who did not complete the course is of interest in this study. A comparison of pre-attitude scores of these students and of students completing the course might provide useful information.

Other questions arising from the study concern the relationship of pre and post attitude scores and final course grade, and of attitude

change and final grade. Another study might investigate whether the use of the historical items affects course grades as well as attitudes.

THREE LEVELS OF DIFFICULTY IN VERBAL ARITHMETIC PROBLEMS. Nesher, Pearla. Educational Studies in Mathematics, v7, pp369-388, December 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by W. George Cathcart, University of Alberta.

1. Purpose

The purpose of this study was to determine the effect of the number of steps (STEPS), the presence of superfluous numerical information (SUP. INF.), and the presence of a verbal cue (V.CUE) on performance in verbal problem solving.

2. Rationale

A review of the literature indicated two approaches to problem solving, the "translation " approach and the "structural variables" approach. The latter approach, advanced by Suppes, Jerman, and others, was adopted for this study. Some limitations of the translational approach were advanced presumably as a rationale for adopting the structural variables approach.

3. Research Design and Procedure

Four story problems were chosen. Two of these involved continuous material and partitive division and two involved discrete objects and measurement division.

Eight versions of each question were written to account for all possible combinations of the independent variables:

X_1: STEPS (1 or 2)

X_2: SUP.INF. (present or absent)

X_3: V.CUE (present or absent).

Other variables controlled included:

(1) length of question	(22 words in the original Hebrew)
(2) required operation	(division)
(3) kind of story	(4 stories in 2 categories)
(4) range of numbers	(natural numbers < 100; 1-digit divisor, 2-digit dividend).

Seven null hypotheses were tested. These stated that there was (were) no significant

(1) differences among four questions used;

(2) difference between steps (1 or 2 steps);

(3) difference between the presence or absence of superfluous information;

(4) difference between the presence or absence of a verbal cue;

(5) two-way interactions;

(6) three-way interactions; and

(7) differences among five sample schools in terms of the performance on the problems.

An eighth hypothesis speculated that the difficulty of the questions could be explained primarily by the superfluous information variable, then by the steps variable, and least effectively by the verbal cue variable.

The sample consisted of 800 subjects 13-15 years of age. One hundred sixty subjects were selected from each of 5 junior high schools each with a "quite different population". Three schools were Jewish and two were Arabic.

Each subject was asked to answer only 4 of the 32 problems used. The problems were presented so that each subject answered a problem involving each of the 4 story settings with each problem representing a different combination of variables X_1, X_2, and X_3.

A four-way ANOVA was performed using a 4(question) x 2(STEPS) x 2(SUP.INF.) x 2(V.CUE) design. There were 3200 problem solutions (800 subjects x 4 problems) with 100 in each of the 32 cells. The criterion for each problem was simply whether or not the final answer was correct.

A separate one-way ANOVA was used to test differences in means among five schools and a multiple classification analysis was used to determine the proportion of variance explained by variables X_1, X_2, and X_3.

When no significant differences were found among the 5 groups, the subjects were pooled for the other statistical analyses.

4. Findings

A significant 3-way interaction was found involving QUESTION X STEPS X SUP.INF. The primary source of the interaction was the combination

of question 2 (a division of cloth to make equivalent suits) x 1 step x presence of superfluous information.

Significant main effects were found for QUESTION, STEPS, AND SUP. INF. but not for V.CUE.

From the multiple classificiation analysis the largest proportion of explained variance was attributable to STEPS, followed by SUP. INF., and lastly by V.CUE. The hypothesized order was SUP.INF., STEPS, and V.CUE.

5. Interpretations

The investigator concludes that more research is needed into the relative contribution of the independent variables to the difficulty of problems. This conclusion was reached after a separate stepwise regression analysis tended to support the hypothesized order of contribution whereas the original multiple classification analysis did not.

The study provides for a contextual effect on problem solving due to superfluous information and the investigator suggests that research which treats this variable as lexical and structural may prove fruitful.

Question 2 behaved quite differently from the other three questions and the investigator advances some reasons for this.

Critical Commentary

The topic chosen for investigation is certainly an important one. Problem solving is a basic skill that teachers have always found difficult to teach. However, this study does not advance our knowledge of either the problem or its solution. We already know that two-step problems are more difficult than one-step problems and that the presence of superfluous information makes problem solving more difficult for children. There was no logical rationale give for conducting the study.

The criterion in this study was simply the product. Would not research into the processes children use to solve problems be much more helpful to teachers and researchers in mathematics education?

The researcher took considerable care to control some important intervening variables. However, the research report lacked explicit information on some points that could affect the internal validity of the study. For example, we are not told how the 5 schools were selected nor how the 160 subjects from each school were chosen. Also, no information is given on how the data were collected. How were the problems presented, to individuals or to whole classes? Were they presented individually on cards or were the four problems given on a single sheet?

Were the problems timed? Were all the data collected at the same time of day or did some subjects have a fatigue disadvantage? Furthermore, no mention was made of IQ, which is an important variable in problem-solving success.

The reluctance of the researcher to accept the results of the multiple classification analysis suggests a possible experimenter bias.

THE OBJECTIVES FOR TEACHING MATHEMATICS IN THE JUNIOR HIGH SCHOOL AS PERCEIVED BY PARENTS, STUDENTS, TEACHERS, AND PROFESSIONAL EDUCATORS. Olson, A. T.; Freeman, E. Alberta Journal of Educational Research, v22 n1, pp52-58, March 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Thomas E. Rowan, Montgomery County Public Schools.

1. Purpose

To examine the degree to which students, parents, teachers, and college mathematics educators agreed upon the objectives which are important in mathematics education at the junior high school level.

2. Rationale

A great deal has been said and written about goals and objectives for mathematics programs during the past fifteen years. Most of these statements have come from mathematics educators. The question of discrepancies between the goals set by educators and those held by the general public will probably be of increasing interest as the trend toward accountability and minimal competencies continues. This study considers whether the junior high school mathematics program is being pulled in many different directions by the various groups involved.

3. Research Design and Procedure

A list of fifteen objectives for the junior high school mathematics program was selected from the literature. They were written in broad language, calling for value judgments by any reader. This list was given to students, teachers, parents, and college mathematics educators. Each respondent was asked to rank the objectives in order of importance. The resultant rankings were analyzed to determine between-group differences.

The study was conducted in three rural counties of Alberta, Canada. It involved 420 ninth-grade student questionnaires, of which 80.5% were returned; 420 parent questionnaires, with 57.6% returned; 39 junior high school mathematics teacher questionnaires, 87.2% of which were returned; and 9 college mathematics educator questionnaires, with 88.9% returned.

Consensus rankings were obtained by summing the ranks of each objective within groups. The Kendall Coefficient of Concordance W was calculated for each group to ascertain that the summing procedure had not masked large within-group differences. The group results were compared using the Mann-Whitney U test to determine significant differences on specific objectives. Finally, the Contingency Coefficient C was computed to determine the degree of association between the groups and the rankings.

4. Findings

The consensus rankings by groups can be seen in Table 1.

TABLE 1

CONSENSUS RANKINGS BY GROUPS

Objective	Teachers' Ranking	Students' Ranking	Parents' Ranking	Educators' Ranking
1. Fundamental processes	4	2	2	7
2. Mathematical concepts	2	4	4	2
3. Process skills	1	3	3	1
4. Problem Solving	3	5	5	3.5
5. Confidence in ability	6.5	5	5	5
6. Mathematics in daily life	5	1	1	13
7. Mathematics in science and technology	10	8	10	15
8. Mathematics and the physical world	11	11	12	11
9. Structure	8	7	8	6
10. Appreciation	14	10	9	8
11. Critical thinking	6.5	9	7	14
12. Mathematics a human activity	15	14.5	15	12
13. Creativity	13	12	14	9
14. Precise language and symbolism	12	13	11	10
15. Enjoyment	9	14.5	13	3.5

N for educators = 8
N for teachers = 36
N for parents = 242
N for students = 338

Significant ($p \leq .05$) differences were found for pair-wise comparisons between: college educators and teachers (objectives 6, 10), teachers and parents (objectives 1, 6, 15), parents and students (objectives 3 and 13), parents and college educators (objectives 1, 6, 7, 15), and teachers and students (objectives 2, 3, 4, 7). The Mann-Whitney U test was the statistic used to determine these differences.

Finally, the Contingency Coefficient C was computed for the groups on each objective. Significance ($p \leq .05$) was found on six of the fifteen objectives.

5. Interpretations

Students and parents seemed quite similar in their perceptions of the objectives. College mathematics educators were appreciably different in their views. Junior high school teachers were somewhere in between, probably closer to the students and parents than to the educators.

The procedure used in this study seems potentially useful. With regard to the objectives, it seems that substantive evidence has been introduced in an area where opinion, bias, intuitive notions, et cetera, generally ruled the day.

Critical Commentary

The importance which can be attached to differences between goals ranked by students, parents, teachers, and college mathematics educators is questionable. Perhaps the most important differences found are those between the teachers and the college educators. The authors seemed to feel that this was an important difference, although they did not single it out as the most important. Unfortunately, the size of the sample of college educators was so small ($N = 8$) that no generalization of the result would be possible. It may, though, have some significance for the region in which the study was made.

With regard to the rankings done by parents and students, one must always consider the question of whether they understood the objectives. The authors said that general language was used and that examples were given to aid interpretations. On such a questionnaire there is no way to ascertain the level of misunderstanding that might have occurred. Comparing results between such obviously different groups as parents and college educators is very questionable. If one looks at the objectives it can be seen that they are a mixture of social skills, mathematical end products, means to an end, method-related objectives, et cetera. To expect students or parents to see the significance of each of these is perhaps asking too much. A better use of the parent and student data might be strictly for relative levels of perceived importance within those groups rather than for comparison with teachers or educators.

Finally, it should be noted that the article, as printed, seemed to have a number of unnecessary inconsistencies. These may have been editorial problems. The data in Table 1 show two rankings of 5 for students and parents (objectives 4 and 5). One can only assume these should have been 5.5 in each case. In presenting the results of the Mann-Whitney U test, the student ranking of objective 3 was cited as 4 while in Table 1 it appears as 3. Were some figures reversed? Which ones? Finally, the results of the analysis using the Contingency Coefficient C are presented almost without comment. It is noted only that there is a significant level of association ($p \leq .05$) on six of the fifteen objectives. The importance (or lack thereof) of that finding is not mentioned. As a matter of fact, if the significance level was $p \leq .05$, then the table presented in the article seemed to show seven significant contingency coefficients, since for objective 4 the level of significance was $p = 0.05$.

"NEW MATH" IMPLEMENTATION: A LOOK INSIDE THE CLASSROOM. Price, Jack; Kelley, John L.; Kelley, Jonathan. Journal for Research in Mathematics Education, v8 n5, pp323-331, November 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Otto C. Bassler, George Peabody College for Teachers.

1. Purpose

To investigate the actual classroom practices used in second- and fifth-grade mathematics classes in the United States.

2. Rationale

Within the recent past there have been many suggestions for modifying the topics taught and the way mathematics is taught in the elementary school. There has, however, been little research pertaining to how these recommended changes are being accepted and implemented by the classroom teacher.

3. Research Design and Procedure

A questionnaire was developed by the authors, modified by a panel of experts, and pretested on a group of practicing teachers. Ten questionnaires were sent to a random sample of 300 supervisors from more than 800 listed as mathematics supervisors by the National Council of Teachers of Mathematics. Each supervisor was directed to select randomly five second-grade and five fifth-grade teachers who were to complete the questionnaire.

Return of postcards from the supervisors indicated that 191 (64%) distributed the questionnaires to teachers. Data were obtained from 1220 teachers who completed questionnaires. This was a response rate of 64% if it is assumed that only those supervisors who returned postcards contacted teachers. If this assumption is not made, the return rate was 41% of the potential sample of teachers.

4. Findings

Results were summarized in several areas. The data were reported as percentages of teachers responding to items; no statistical comparisons were given. It was stated that differences between respondents by geographical areas, socioeconomic status of the school, and grade level taught were small.

(a) Characteristics of Teachers: Over 90% of the second- and 70% of the fifth-grade teachers were women; 53% had been

teaching ten years or less; and 56% were 40 years of age or younger. Teaching of mathematics was found very interesting by 65% of the teachers--in fact 45% preferred it to the teaching of reading or social studies while only 11% liked teaching it least of these three subjects. Most teachers (53%) believed that present students were doing better than past comparable classes while 20% thought present students were doing less well. With regard to prior mathematics training, 88% had at least one year of high school algebra; 70% had at least one year of high school geometry; 63% had at least two mathematics courses in college; and 48% had at least two mathematics education courses in college. Most (84%) of these teachers do not belong to any professional mathematics organization.

(b) Objectives and Assessment: State or locally published objectives for mathematics were available to 83% of the respondents and most (63%) used them in their teaching. Some form of state or local mathematics assessments was available to 77% of the teachers and 43% indicated that they based their teaching on the results of these assessments.

(c) Textbooks and Topics: One textbook, as the single source of mathematics information, was used by 56% of the teachers; 26% more used mainly one textbook from two or more available textbooks; and only 7% used no textbook at all. Books emphasizing skills over concepts were preferred by 42%; 49% preferred equal emphasis on skills and concepts; and 3% preferred emphasis on concepts over skills. Texts were followed closely by 53% of the teachers but over half of them did not require students to read as much as two pages in every five. There was little or no treatment in texts or supplementary sources for the topics of metrics (66%), probability (64%), and graphs and statistics (52%) as indicated by the teachers. Hence fewer than 5 periods per year were spent on these respective topics by 61%, 75%, and 56% of the teachers.

(d) Class Time: One-half hour or more is spent teaching mathematics in 90% of the classrooms sampled. The weighted average time-per-day for all respondents was 43 minutes. Of this average time, 43% was spent in written or seat work; 36% in discussion or explanation; and 21% in other activities.

(e) Teaching Methods: Whole-class instruction was used most of the time by 40% of the respondents but only 6% said they had never tried individualizing. Grouping by ability was favored by 84% of the teachers. Most teachers used laboratory experiences only occasionally (72%), had never used computer-assisted or computer-mediated instruction (81%), had never used instructional television (78%), had never used hand calculators (83%), and had never tried any type of team teaching (68%).

(f) In-Service: Mathematics-related workshops or courses had been taken by 32% of the respondents during their present school year (1975), but another 30% had no in-service training since 1970 or earlier. Observation of other teachers had never been done by 28% of the sample and 74% had observed other teachers at most 4 times; however, 64% thought that such observations would improve their teaching.

5. Interpretations

The major conclusion is that mathematics teachers and classrooms have changed less in the past 15 years than had been supposed. Generalized descriptions of teachers and classes are provided. "Median" teachers were described as female; under 40; having 10 or less years experience; prior coursework includes two semesters of high school algebra, two of high school geometry, two college mathematics courses, and one college course in mathematics education. The "median" teacher is female, does not belong to any association of mathematics teachers, and has had limited in-service experience in mathematics education. The "median" class is self-contained; it lasts about 43 minutes, of which about half the time is spent in written work. A single text which is followed closely is used in whole-class instruction. Almost none of the concepts, methods, or "big ideas" of modern mathematics programs are used in the classroom.

Other conclusions were:

(a) If there are declines in mathematics test scores, only a small decline can be attributed to "new mathematics."

(b) Specialized mathematics teachers are a little better trained, have a more positive attitude toward teaching mathematics, and seem to make more use of concepts and processes than teachers in self-contained classes.

(c) Even if increased funding and assistance have been available in schools of lower socioeconomic status, little change in teaching method has resulted.

Critical Commentary

This study provides useful information about second- and fifth-grade classroom teachers and the activities that are going on in their classrooms. The results are disturbing in that they show little impact of curricular innovations on actual classroom practice.

As pointed out by the investigators, it is conceivable that the method of selecting the respondents through supervisors introduced a bias in the results, since not all school districts have mathematics

supervisors. A more likely cause of bias are the low return rates from supervisors and then from teachers. Both of these biases, however, would tend to produce results that are more optimistic than actual classroom practice.

Since actual data and number of respondents in subcategories were not reported, it is impossible to verify the accuracy of the conclusions. The authors were careful to state the conclusions in a manner that seemed to be warranted, as well as implying that they were only tentative in nature.

MODELING EFFECTS AND DEVELOPMENTAL CHANGES IN DEALING WITH A FORMAL OPERATIONS TASK. Ronning, Royce R. American Educational Research Journal, v14 n3, pp213-223, Summer 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward Esty, National Institute of Education, Washington, D.C.

1. Purpose

The purpose of the study was to determine the effect of age and the effect of providing a model of correct strategy on children's performance on a verbal task at the formal operations level.

2. Rationale

Although much previous research has considered the transition from preoperational to concrete operational thought, comparatively little research has examined the transition to formal operational thought. Hence little is known about how children cope with formal operational tasks that are too difficult for them, even though many tasks that children encounter apparently demand formal operational thought.

The present investigation is an extension of two lines of previous research: (1) that of Neimark and Lewis (1967) and Eimas (1970) on the development of strategies in binary problem-solving tasks; and (2) that of Laughlin, Moss, and Miller (1969), Denney (1972), and Denney, Denney, and Ziobrowski (1973) on the effects of modelling the correct strategy for solving a problem.

3. Research Design and Procedure

A total of 64 subjects from a middle-class community were used, half boys and half girls. At each of four grade levels (grades 1, 3, 5 and 7), four boys and four girls were randomly assigned to a model condition and to a no-model condition.

Each student was tested individually by a female investigator. The task in each case was the same: to determine, by asking no more than seven questions, a number from 1 to 100 that the experimenter had previously chosen from a table of random numbers. Each subject had three chances to play the "game".

Half of the subjects, those in the model condition, were asked to listen carefully to a tape recording of a boy playing the game using a strategy that guarantees success within the allotted seven questions--by asking a series of yes-no questions that successively eliminate about half of the remaining numbers (e.g., "Is it more than 50?" "Is it less than 25?")

Two strategy scoring systems were used, one more lenient than the other. In Scoring System I (SSI), 1 point was given if any one of the child's seven questions eliminated more than one number; e.g., "Is it a prime?", "Is it less than 95?" (Such questions are called "constraint-seeking.") A child's maximum score under this system was thus 3, since the subjects were given three chances to play the game. The Second Scoring System (SSII) awarded 1 point if two consecutive questions were constraint-seeking. As in SSI, the maximum score for each child was 3. The number of correct solutions was counted also, yielding a third score with a maximum of 3 points for each subject.

The three scoring systems were analyzed using three univariate ANOVAs, even though some of the assumptions underlying that procedure were violated.

4. Findings

Of the two strategy scoring systems, only SSII is discussed in the paper, because, according to the investigator, the results for the two scoring systems were "the same". Means for SSII were as follows:

Grade Level

	First		Third		Fifth		Seventh		
Condition:	M	NM	M	NM	M	NM	M	NM	Sex $\bar{X}$
Males	0	0	1.50	1.00	3.00	2.25	2.50	2.25	1.56
Females	0	0	1.50	0.75	2.75	0.25	2.75	2.25	1.28
$\bar{X}$ Condition	0	0	1.50	0.87	2.87	1.25	2.62	2.25	
$\bar{X}$ Grade	0		1.19		2.06		2.44		
$\bar{X}$ Age (years)	6.1		8.1		10.3		12.4		

There was a significant effect for grade: $F(3,48) = 21.33$, $p < .0001$. There was also a significant main effect for condition: $F(1,48) = 8.68$, $p < .0005$. Neither a significant main sex effect nor significant interaction effects were found.

Considering the number of correct solutions (i.e., the number of times the correct number was determined within the seven questions), the following mean scores were obtained: Grade 1 -- 0.06; Grade 3 -- 0.75; Grade 5 -- 0.87; and Grade 7 -- 1.00. (Recall that the maximum score is 3.00.) Only the effect for grade was significant here: $F(2,48) = 4.05$, $p < .01$.

The investigator states that "In view of the low frequency of success in the first grade as compared to the other grades, the significant effect is assumed to be largely between grade one and the remainder of the sample."

Because some students asked questions like "Is it 1000?", a tabulation of "out-of-bounds" questions was made. Seven different children, all at the first-grade level, asked a total of 37 such questions.

5. Interpretations

The investigator discusses the task from a Piagetian viewpoint, describing how one might expect children of pre-operational, concrete operational and formal operational stages to respond. He then discusses the actual results, noting in particular that (1) the first-grade children's performance was uniformly low, with no difference between the model and no-model conditions; (2) there is a large difference between the model and no-model conditions at the fifth-grade level, indicating that fifth graders can use the strategy presented but are unlikely to think of it themselves; and (3) there is relatively little difference between the two conditions at the seventh-grade level. The results are seen as fully consistent with Piagetian theory.

Finally, "While both the developmental and model effects are powerful in terms of strategy acquisition, the relatively small number of correct solutions even at the oldest age examined suggests the need for examination of other variables such as instructions, use of examples, and reinforcement/motivational conditions."

Critical Commentary

One piece of information that was omitted is crucial to one's interpretation of the results of this study: no mention is made of the point during the school year at which the experiment was conducted. This is especially important to know for the first graders, because the first-grade year typically includes considerable work with initial concepts of order relations in the whole numbers, and, of course, work with two-digit numbers. The fact that first graders did so poorly is not necessarily due to their inability to grasp a useful strategy: it may simply be that they did not know what "less than" or "more than" or "between" meant. Thus the experimenter's instructions and the tape-recorded model performance may have been completely incomprehensible to some of them. (In this respect this study provides yet another example of the value of an interdisciplinary approach, for any first-grade teacher would have suggested that the students' knowledge of order concepts be assessed and described in the paper.) The possibility that the first graders, who were of mean age 6.1, did not know enough about order relations for the task to make any sense makes all the more interesting the fact that there were such small difference among the third, fifth, and seventh graders on the number of correct solutions obtained.

Further information and discussion of a few other points would have been helpful; for example, the fact that the fifth-grade boys did better (in SSII) than the seventh graders of either sex. Was this a fluke of the sample, or is there some underlying reason? It would have been useful too if the investigator had supplied more information about the variation in children's performance. For example, how many children always followed a constraint-seeking strategy? How many got the correct solution in all three trials?

It should also be mentioned that the investigator's constant reference to "the best" strategy is open to some question. What is "best" depends to some extent on the payoffs involved. In the 1 to 100 game used here, if there is any payoff whatsoever for determining the number in fewer than seven questions, then the strategy described is not "best". One can still guarantee getting the number within seven questions and yet have a chance of guessing it earlier if one first asks "Is it larger than 64?" and then, if the answer is affirmative, "Is it larger than 96?"

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Expanded Abstract and Analysis Prepared Especially for I.M.E. by Charlotte Wheatley and Grayson Wheatley, Purdue University.

1. Purpose

The purpose of these studies were to study calculator-assisted learning of mathematics by elementary school pupils with attention to computation, problem solving, motivation, instructional methods, and applicability to certain topics. Some of the questions asked were:

(1) What is the effect of calculator availability on the motivation of young children?

(2) Can five-year-olds profit from the use of calculators?

(3) What topics can be taught more effectively with a calculator?

(4) What implications does the calculator hold for problem solving?

2. Rationale

Because the small electronic calculator is becoming inexpensive and available, the authors thought it important to explore its impact on children's learning of mathematics. These studies were not conducted to test hypotheses but to generate hypotheses and to explore the feasibility of calculator use with primary school pupils in learning mathematics.

This article reports the results of a set of five feasibility studies. None of the studies employed experimental controls or comparative statistics. The conclusions are based on observational data and pupil reactions to calculator use.

3. Research Design and Procedure

In a series of five exploratory investigations, pupils of aged five to nine used calculators in learning mathematics. The number of class sessions varied from eight to thirty-two. In experiment one, two groups of six-year-olds studied arithmetic, one with a calculator and the other without, in 15 lessons of 20 minutes each. The children in the non-calculator group made extensive use of manipulative materials. A similar comparison of calculator impact was made with five-year-olds in 30 lessons. Seven-year-old children solved cost problems encountered in shopping

(e.g., amount of change when several items are purchased). This unit was eight lessons long. In a fourth study, five-year-olds learned to use a calculator either by free exploration or exposition. In the discovery group, the pupils learned by trial-and-error pushing of keys, while the exposition group was explicitly taught the function of keys. In a feasibility study with five third- and fourth-grade children, a variety of topics that might effectively be taught with a calculator were introduced in 32 class meetings of 30-60 minutes each.

4. Findings

The authors report their observations of the differences in the calculator and non-calculator groups. No performance comparisons were made. Five- and six-year-old children were observed to be highly motivated to study mathematics (typical grade-level topics), while control group pupils were not motivated by the mathematics or the use of manipulatives. The high interest displayed by the calculator group was sustained over the entire period of use. The children using calculators were less distracted, displayed longer attention spans, and worked independently for long periods of time. Contrastingly, the non-calculator group did not display interest, were confused by the manipulatives, showed little imagination, and waited for teacher direction.

The five-year-old children preferred a desktop calculator with larger display and keys. A group of five-year-old children taught by expository methods to use a calculator could solve presented problems, while a group allowed to explore calculators could not. Seven-year-old children were more highly motivated and more successful in problem solving than children not using calculators. Eight- and nine-year-old children showed marked mathematics achievement gains over a 10-week period while using calculators; some had more than a year grade-level gain in computation, concepts, and applications. The calculator was reported as being highly successful in motivating and assisting these children in learning mathematics.

5. Interpretations

The authors conclude that the learning of mathematics is facilitated by the use of calculators. Specifically they suggest that:

(1) place value (whole numbers and decimals), negative numbers, decimals, and factoring can be taught more effectively with a calculator,

(2) problem-solving skills can be greatly enhanced through use of calculators,

(3) the standard mathematics curriculum can be expanded to include use of numbers of greater magnitude,

(4) estimating skills, negative numbers, and decimals can be introduced at a much earlier time,

(5) computational skill may be enhanced through calculator practice.

Critical Commentary

This article reports five studies which were designed to explore the effects of calculator use in learning mathematics. The results are purely observational with no attempt to determine achievement differences. There is a definite place for exploratory studies in mathematics education research. Properly designed teaching experiments can lead to the identification of hypotheses for further study. They may allow the experimenter to understand the thought patterns of children. Results of teaching experiments can also lead to the development of curriculum materials. However, this report contains insufficient information for the interpretation of the results stated. In the first study, no details are given on (1) the number of subjects, (2) the size of instructional groups (Was the instruction in small groups?), (3) method(s) of instruction, or (4) the number of calculators per group (Did each pupil have a calculator?). Yet the authors conclude, based on observation, that the calculator-assisted learning was vastly superior. Care must be exercised in interpreting and utilizing findings based solely on the impressions of the experimenters. Additional detail would have provided the reader with the necessary information to interpret the conclusions.

While the observational results favor the calculator groups, no comparative performance data were reported. It is possible that the non-calculator group, appearing less motivated, may have achieved more. No assertion is being made that this was, in fact, the case in this study, but the possibility must be considered.

The low interest level reported for pupils using manipulatives is not in agreement with numerous studies which have established the motivational value of manipulative materials. One is led to suspect a teacher-bias effect against the non-calculator group.

The study with eight- and nine-year-old children had only five subjects. The authors chose to report achievement test results on only three of these five subjects. Why only these three? What were the scores for the other two? The practice of selecting data to report is highly questionable. The number of subjects in the other four studies is not reported.

The study comparing "discovery and exposition" teaching strategies was poorly conceived. To give five-year-old children calculators without any direction and expect them to "discover" calculator logic is unreasonable. While it may not be necessary to teach explicitly each key function, at least children need suggested activities to incorporate the

calculator as a tool in their thinking. A better test of the discovery approach would be to teach children to use calculators and then let them explore.

It is quite clear that the authors were very impressed with the advantages of calculator use in learning mathematics. While the calculator may be a valuable new instructional aid, the total effect of calculator-assisted instruction must await more careful evaluation. We do not often find panaceas for the problems of education; it is doubtful that the calculator is one.

SELF-PACED CALCULUS: A PRELIMINARY EVALUATION. Struik, Ruth R.; Flexer, Roberta J. American Mathematical Monthly, v84, pp129-134, February 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Harold L. Schoen, University of Iowa.

1. Purpose

To compare a traditionally taught calculus course with a self-paced course based on students' performance on a final examination.

2. Rationale

No previous research or theoretical framework is cited.

3. Research Design and Procedure

A first-semester, five-credit calculus course at the University of Colorado was offered in both a self-paced instructional mode and a traditional lecture-recitation mode. One hundred five students chose the self-paced section while 130 elected the traditionally run section.

The students electing self-pacing could select any of three differently paced presentations--fast, medium, or slow. Later this was modified to one medium and two slow presentations. Classes met for four 50-minute sessions per week. In addition, one and one-half days per week were provided for taking 12 quizzes and the midterm examination. Students received frequent individual help, were given up to five chances to pass each quiz, and had the option of taking an early final examination.

"Those electing traditional instruction attended three 50-minute lectures and two 50-minute recitations weekly. Quizzes and examinations were administered, graded, and returned in a traditional manner."

Cumulative grade point averages (GPA) and American Testing Program (ATP) mathematics examination scores were collected for all students. The dependent variable was the student's grade on a common final examination. A two-way analysis of covariance model (method x sex) using GPA and ATP as covariates was employed. In addition, a three-way analysis of variance (ATP x method x sex) was run primarily to test for ability x method interaction.

4. Findings

Group means and numbers of students are given in the table. The difference in means between the treatment groups was significant beyond the 0.01 level ($F_{1,148} = 8.8$; both GPA and ATP taken into account). No other effect was significant.

Table. Means of Final Examination

		Sex		Raw Means
		Male	Female	
Method	Self-Paced	$\bar{X}$ = 172.9 n = 41	$\bar{X}$ = 172.3 n = 16	$\bar{X}$ = 172.7 n = 57
	Traditional	$\bar{X}$ = 149.4 n = 72	$\bar{X}$ = 164.5 n = 25	$\bar{X}$ = 153.3 n = 97
	Column Means	$\bar{X}$ = 157.9 n = 113	$\bar{X}$ = 167.5 n = 41	

Interactions between sex and method and between ability and method were not statistically significant.

No students failed in the self-paced section. In order to test for a possible spurious depression of the traditional group mean due to a 9.8% failure rate, the data were also analyzed after deleting the data from failing students. Again, there was a statistically significant ($p < 0.025$) difference in means favoring the self-paced group (means were 172.7 compared to 165.9 with Ns of 57 and 84, respectively).

5. Interpretations

"The conclusion to be drawn from this analysis is quite clear: students in the self-paced group did significantly better than students receiving traditional instruction."

Lack of random assignment of subjects, possible scoring differences on the final examination, and the possible confounding variable of teacher effect are cited as flaws in the study. Nonetheless, two implications for departments of mathematics were drawn. First, self-paced sections should be made available as an option for students with full awareness that more instructional time is involved. Second, departments of mathematics should recognize females as a potentially rich source of capable mathematics majors.

Critical Commentary

This study reports the results of an evaluation of a particular approach to instruction in the calculus. The approach seemed to work at the University of Colorado with their instructors and students. It might be a viable approach for other institutions. In my opinion, not much more can be concluded from the study for the reasons which follow.

The authors list three weaknesses in the design: use of volunteers for subjects, possible scoring differences in final examinations, and the teacher effect. In fact, these three weaknesses make interpretation of the results impossible. Since the students in the self-paced class volunteered to be there, at most one could conclude that students who volunteer to be in a self-paced class do better than non-volunteers in a traditionally run class. However, the possible differences in teachers cannot be excluded as an explanation for the results. Worst of all--because it could have easily been corrected--the possible systematic difference in the scoring of the examinations appears to eliminate any chance of a meaningful interpretation.

Given that the data presented and analyzed in this study are neither generalizable nor easily interpreted for this sample, what is the value of the article? It seems to me that university mathematics instructors may be interested in trying this approach--namely, traditionally run classes with a self-paced section available as a student option. Such a scheme has support from previous research although it is not cited by the authors (see Schoen, 1977). Unfortunately, the authors do not describe the self-paced method in any detail. This makes it impossible for an interested instructor to replicate it.

In conclusion, I wish to emphasize several points:

1. I agree whole-heartedly with the authors' statement that ". . . self-pacing should be available only to those who elect it." This is supported by previous research findings.

2. The "self-paced" method in this study probably was not the learning packet, bit-by-bit approach, but consisted of teacher-centered presentations geared at different rates with individual tutoring available upon request.

3. With regard to the individual tutoring, "It may be that any method that utilizes so large an amount of instructional time will give similar results."

An attempt should be made by journals and authors to distinguish between research articles in which the rationale, design, and procedures make meaningful interpretation of data possible and those articles describing an approach to teaching that was apparently used successfully somewhere. In this second type of article, the space used for data analysis could better be allotted to a detailed description of the teaching method, materials, management system, et cetera. This would allow the approach to be adopted or adapted by another teacher or institution. Thus the practitioner in search of teaching ideas could gain useful information while not being misled by seemingly "hard" data which, in fact, rest on very weak foundations.

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AN EVALUATION OF THE MATHEMATICS-METHODS PROGRAM INVOLVING THE STUDY OF TEACHING CHARACTERISTICS AND PUPIL ACHIEVEMENT IN MATHEMATICS. Thornton, Carol Dodd. Journal of Research in Mathematics Education, v8 n1, pp17-25, January 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by James E. Schultz, The Ohio State University

1. Purpose

The purpose of the study was "to explore a technique for evaluating (preservice elementary) teacher education programs in terms of teaching competencies and, in particular, to explore that technique as applied to the Indiana University Mathematics - Methods Program" (an integrated content-methods program referred to henceforth as "MMP"). Two major questions were identified:

(a) Is there any difference in performance on certain teacher variables by preservice teachers (PSTs) trained by the MMP and those trained by alternative programs?

(b) Is there any relationship between the achievement of the pupils taught by the PSTs and the performance of the PSTs on these teacher variables?

2. Rationale

In an effort to probe the effectiveness of the MMP and the relationships among pupil achievement and teacher characteristics, the study extended a teaching performance test paradigm over certain "product" variables described by Popham to include certain "process" variables. The process variables identified were (a) clarity in developing a mathematical idea, (b) questioning techniques, and (c) involvement of pupils.

3. Research Design and Procedure

The PST subjects were undergraduate elementary education majors at Indiana University. Ten students were randomly selected from each of three programs. Students in the MMP format had teaching experiences which related to their mathematics instruction in content and methods over two semesters. Students in Contrasts I and II had previously completed the required mathematics courses for teachers and were enrolled in mathematics methods courses. Those in Contrast I had no field experience in conjunction with their course and those in Contrast II had participated regularly in field experiences of a general nature - - not restricted to the teaching of mathematical topics. The pupils taught by the PSTs were 120 third-grade pupils.

PSTs were given instructional objectives 3 days before teaching two half-hour sessions introducing equivalent fractions to a group of 4 third-graders on consecutive days. Teaching sessions were videotaped for evaluation by 4 graduate students who had been trained as observers. Each tape was randomly assigned to two observers who used frequency counts and ratings to assign values to the clarity, questioning, and pupil involvement variables.

Prior to the actual teaching, a test on equivalent fractions was administered to each PST and an IQ test and a fraction inventory were given to the third-grade pupils. These latter two measures were used to adjust pupil posttest scores for initial differences.

4. Findings

A sequence of multivariate, univariate and Tukey tests led to the reporting of several significant differences. The ranking of the groups for each of the five variables (from low to high) is summarized below, where an underline joining 2 variables indicates no significant difference at the .05 level. Here "M" represents the MMP group and "I" and "II" represent Contrasts I and II respectively.

	Low		High
PST Test	I	II	M
Clarity	I	II	M
Questioning	I	II	M
Involvement	I	II	M
Pupil Score	I	II	M

Significant ($p < .01$) Pearson product moment correlations were found between each pair of variables chosen from clarity, questioning, pupil involvement, and pupil score. The clarity vs. pupil score correlation was .96. Reliability measures for instruments and coder consistency exceeded .83 in all cases.

5. Interpretations

The investigator stated the following conclusions:

(a) The procedures used in this study should be considered a viable technique for evaluating segments of a teacher education program.

(b) The three teacher variables, Clarity, Questioning, and Involvement, are strong correlates of pupil achievement.

(c) The MMP appears to have promoted competency associated with the variables Clarity, Questioning, Involvement and Pupil Score more effectively than the other two programs of the study.

(d) There is an indication that regular, planned school experience in conjunction with the mathematics preparation of preservice elementary teachers may have an impact on teacher competency to produce mathematical learning in children.

However, the investigator disclaimed the ability of the study to draw definite conclusions comparing the programs with regard to the variables studied or other variables and urged further studies focusing on field experiences, since the sample in this study was small, the school population was atypical, and the teaching lessons involved only one topic.

Critical Commentary

For the most part, the article reported a useful, carefully done study. The investigator usually was aware of potential difficulties and conscientiously averted them or acknowledged instances where they could not adequately be controlled. Nevertheless several possible flaws exist in addition to those identified by the investigator.

Though PST subjects were randomly selected from the three teacher training programs at Indiana University, how were students assigned to these three programs in the first place? The scores of the third-graders were adjusted for initial differences, but initial differences which may have existed among the PSTs in the three groups were not addressed.

We know that PSTs in the MMP group were currently studying mathematics (since the MMP is an integrated approach); yet we are told that those in the other two programs had previously completed the required mathematics courses. Would this not introduce retention of the mathematics as a contaminant? Differences in scores on the PST equivalent fractions test (though not statistically significant) did favor the MMP group, possibly supporting the view that retention was a problem in Contrasts I and II. The fact that the time to prepare the lessons was only 3 days might also favor the mathematically more current group (MMP).

The coders were graduate students. Were they affiliated with the instructional approaches in any other way? Were they blind to the treatment groups from which the PSTs were selected? Was experimenter bias introduced by the fact that the investigator was simultaneously a contributing author to the MMP?

The reviewer would like to underscore the investigator's concern for the ability to draw definite conclusions comparing the programs. Assignment of students and teachers to programs and the many factors not within the realm of control obviate facilitating direct comparisons.

The questions raised above should not, however, discount this solid contribution to the literature.

THE CONCEPT OF EXPONENTIATION AT THE UNDERGRADUATE LEVEL AND THE DEFINITIONAL APPROACH. Vinner, Shlomo. Educational Studies in Mathematics, v8 n1, pp17-26, 1977.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Zalman Usiskin, University of Chicago.

1. Purpose

To deal with the role definitions have in algebra when new arithmetical operations are defined in terms of old operations. The paper consists of a study and extensive discussion of this role, with specific examples purposely restricted to exponentiation.

2. Rationale

The process of definition has a very important role in the structure of mathematics. "The student is expected to understand that there is "some difference" between the formula "$a^{-x} = \frac{1}{a^x}$, where x is a whole number," and the formula: "$a^x a^y = a^{x+y}$, where x, y are (for the sake of simplicity) whole numbers" (p. 17). Do students look at exponentiation the same way as some of their teachers tried to teach them? It is important at the college level to know what we can assume about students' backgrounds and their views of mathematics.

3. Research Design and Procedure

The eight questions listed below were administered to 195 mathematics freshman enrolled in a calculus course and 56 upper-level mathematics students at Berkeley. Each question had the same five choices:

(a) a theorem

(b) a law

(c) a fact about numbers

(d) a definition

(e) an axiom

The questions were:

(1) The equality: $\sqrt[n]{a^m} = a^{m/n}$ in:...

(2) Let a,b denote two arbitrary positive numbers. The equation: $(-a)(-b) = ab$ is:...

(3) The equality $(a + b)c = ac + bc$ is:...

(4) The equality $\underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}} = a^m$ is:...

(5) Let a,b denote two arbitrary real numbers. The equation: $(-a)(+b) = -(ab)$ is:...

(6) Let a be any real number different than zero. The inequality $a^2 > 0$ is:...

(7) Let m denote any natural number. The equality: $1/a^m = a^{-m}$ is:...

(8) Let a,b denote two real numbers. The statement 'a > b if and only if a-b > 0' is:...

Questions 1, 4, and 7 were the only ones analyzed. In each of these, the usual order of the sides of the equality was reversed "to make it less familiar to the student" (p. 18).

4. Findings

Table 1

(Freshmen)

Question Number	A theorem	A law	A fact about numbers	A definition	An axiom
1	14.9	9.2	22.6	51.3	2.0
4	21.5	5.1	11.3	52.8	8.2
7	11.8	7.2	12.3	63.1	5.6

Table II

(Upper Undergraduate Level)

Question Number	A theorem	A law	A fact about numbers	A definition	An axiom
1	19.6	3.6	7.1	66.1	3.6
4	10.7	5.4	7.1	75.0	1.8
7	12.5	1.8	5.4	80.4	0

Taking "a definition" as the correct response, 56% of the freshmen and 80% of upper undergraduates had at least 20 of 3 correct responses; 28% and 52% respectively had all 3 correct.

5. Interpretations

Over half of this article is devoted to interpretations. Among these are: "The results are not satisfactory since we are dealing with math majors and the problem is so elementary" (p. 22). The author feels that many of the students see mathematics as describing and revealing the laws and facts of the world of numbers just as science reveals the laws and facts of the concrete world. The idea that mathematicians define operations and other notions does not agree with many students' views. Thus these students call the properties 1, 4, and 7 "laws" or "facts."

The author connects the results to Piagetian notions. There is a question of readiness. Here the readiness is not previous knowledge but a correct approach to the subject matter. "To teach the definitional approach before the student is at the suitable intellectual stage is just useless (although he might pass the exam)" (p. 25). The author concludes that "the definitional approach should be eliminated from the non-major mathematics curriculum" (p. 25).

Critical Commentary

The author perceptively states a number of plausible reasons for the non-unanimity of answers, including high school textbook confusion of definition and theorem; the semantic difficulties surrounding the word "law", particularly as applied to the properties of exponents; the appearance of $a^m = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}}$ in the student's experience before definitions, laws, and facts are distinguished; and the possibility that, in the approach the student used, these were not definitions but postulates or theorems. Given all of these possibilities, the percentages in tables I and II do not seem particularly low. Indeed, they seem to this reviewer to be high!

A study can be no stronger than the instrument it uses. It was helpful to have all questions published as they were given. So I gave this test to my students and talked to them about it. They felt, as I do, that each question could have had many answers. Some of them were familiar with more than one approach to exponents. The author reports no follow-up or interview with any student. Thus there is no test of the major point of this article, that students answered as they did because of a lack of understanding of the definitional approach. In light of the lack of attempt to distinguish causes, there is no rationale for the final conclusion, let alone a justification for the strong terms in which the final conclusion is stated.

Could it be that the author had his opinions formed before doing the study?

THE ASSOCIATION BETWEEN TEACHER PARTICIPATION IN NSF INSTITUTES AND STUDENT ACHIEVEMENT. Willson, Victor L.; Garibaldi, Antoine M. Journal of Research in Science Teaching, v13 n5, pp431-439, 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Edward J. Davis, University of Georgia.

1. Purpose

This study was directed at the question, "Is there any evidence that precollege student cognitive achievement has been increased because of teacher participation in NSF-sponsored institutes? It should be noted that this study was conducted as a NSF-sponsored project.

2. Rationale

The authors make the following argument for a post hoc analysis:

> An experimental comparison between students whose teachers had attended institutes and students whose teachers had not would be optimal. The experiement would require random assignments of teachers to institutes (or not) and random assignments of students to teachers. Since NSF has not followed such a strategy, post hoc comparisons may be confounded by certain demographic and personological differences between teachers who have attended and those who have not attended NSF institutes. All potential factors can never be discounted in a post hoc analysis, but those theoretically most relevant should be dealt with. (p. 431)

3. Research Design and Procedure

A post hoc analysis was performed. The authors identified science (or mathematics) achievement of teachers, and the level of classes to which a teacher is assigned to be the theoretical and relevant threats to examining the relationship between teacher institute attendance and students' academic improvement.

An urban-rural sample of junior and senior high schools was selected for science from Wyoming, South Dakota, and Mississippi and for mathematics from California and Indiana. Urban representation was small. Eighty-one percent of the science and ninety-one percent of the mathematics classes and teachers came from small towns and cities under 50,000 population. Within each school the principal was asked to select randomly one science (or mathematics) teacher and then select randomly one class from this teacher's load. This yielded a total of 346 science teachers and their classes and 211 mathematics teachers and their classes. Each teacher was given an achievement test in the subject area (NTE exams in either Physics-Chemistry-Science or Mathematics). Science students took

a 40-item test taken from the NAEP science test and the mathematics students were given 40 items from the NLSMA item pool. Different 40-item forms were developed for junior high and senior high classes. Not all students took these achievement tests. Each teacher was given instructions to assign randomly attitude, process, and achievement instruments.

From a background questionnaire teachers were classified as having NO, LOW (1 or 2 institutes attended), or HIGH levels of participation in NSF institutes. This placed 36, 36, and 28 percent of the science teachers and 43, 29, and 28 percent of the mathematics teachers in the respective groups.

The procedures above provided the investigators with a means to control teacher achievement and level of class assignments which were identified as obstacles to examining NSF institute participation and student achievement. Teacher achievement on the NTE exams was used as a covariate in analysis of student achievement. The random selection of teachers and classes was used to produce a situation wherein approximately equal proportions of high-, middle-, and low-ability classes appeared in the NO-LOW-HIGH partition of the teachers. The authors state:

> The possible differential assignment of institute attenders to higher-ability classes was examined by testing the independence of NSF participation from the teachers' assessment of the ability group of the class from which the achievement data were drawn (high ability, average ability, low ability, and mixed ability groupings). Also tested within the senior high school science data was the independence of type of class (biology, chemistry, and physics) from NSF participation. The chi-square statistics was used for each test, . . . All chi-square statistics were nonsignificant at $p = .05$, indicating independence of the distribution of teacher assignments by ability grouping, or subject matter in science, from NSF institute participation. (p. 433)

4. Findings

It was reported that:

> The marginal means of student achievement for NSF participation show a consistent trend in the direction of better student performance with increased teacher NSF participation for all four analyses. . . These means are essentially unaffected by adjustment for the covariate, since none of the regressions were significant at $p = .10$. . . . The nonsignificance of the covariate implies that teachers' science ability is not related with their students achievement. (p. 435)

To follow up differences in means scores, two planned orthogonal contrasts were performed on the senior high science scores and two

more on senior high mathematics scores. These contrasts used an F statistic. The first considered the combined scores of students of LOW and HIGH vs. NO teacher-institute participation. The second contrast compared the scores of LOW vs. HIGH participation. Three of these four contrasts had significance at the .01 level. These are reported as suggesting that teacher attendance at institutes is associated with higher student performance than no attendance, and that students whose teachers attended the higher number of institutes (more than 2) did better than students of teachers attending only 1 or 2 institutes.

5. Interpretations

The authors conclude that a real institute effect is present. They prescribe that institute attendance be required of all secondary science and mathematics teachers.

Critical Commentary

This study investigates an important area. In terms of time and money, a great deal is being and has been invested in in-service education. Student achievement is seldom used as a criterion to evaluate in-service programs. It is relevant to do so.

I am left with some questions, however. When principals are contacted is it likely that they will select a science or mathematics teacher (and one of their classes) at random? Or will a principal tend to choose a teacher and a class according to some preconceived criteria in spite of guarantees of non-identification of participants? What about the levels of difficulty of the achievement tests? Were they constructed to reflect the range of cognitive behaviors identified in the NSLMA study (Computation-Comprehension-Application-Analysis)? What about the attitude and process measures? How were they constructed? How did the students perform on them?

Were these my only concerns, I would feel good about this study, However, I must take exception to the authors' conclusions and recommendations. The trend is for students having teachers who participated in NSF-sponsored institutes to have a significantly higher mean score than students having teachers who did not attend institutes. But how much higher are these means? About 1 or 2 points (items) on one 40-item test. With a large sample it is possible for such a small mean difference to be significant. Statistical significance is present but it is questionable whether this difference is meaningful or possessing any practical significance. When one considers the cost of an institute, to both sponsors and participants, a recommendation that teachers be required to attend them, based primarily on gains of 1 or 2 points on one 40-item instrument seems at best premature and at its worst feathering one's nest.

MASTERY OF SIMPLE PROBABILITY IDEAS AMONG G.C.E. ORDINARY LEVEL MATHEMATICS CANDIDATES. Wood, R.; Brown, M. International Journal of Mathematical Education in Science and Technology, v7 n3, pp297-306, August 1976.

Expanded Abstract and Analysis Prepared Especially for I.M.E. by Richard D. Lodholz and Douglas A. Grouws, University of Missouri-Columbia.

1. Purpose

To investigate the apparent difficulty of probability test items on mathematics examination papers taken by the most able 25-30 percent of 15- to 16-year-old English students and to investigate differences in difficulty between sexes.

2. Rationale

An anyalsis of responses to the General Certificate of Education (G.C.E.) Ordinary Level mathematics examinations of 1973 and 1974 by the University of London School Examination Department disclosed an apparent difficulty in probability items relative to the other items on each examination, and an apparent differentiation of responses between sexes. In the 1974 paper, two of the three probability items showed lower percentages correct than 48 of the 60 items; the average percentage difference between boys and girls was 17%, while the average marks for all items differed only by 6% (both favorable to the boys).

According to Wood and Brown, previous studies have shown that probability calculations involving the multiplication law and combinatory operations in permutation calculations are unlikely to be correctly done until the Piagetian Stage IIIB (full formal operations). Probability items involving (1) an elementary event; (2) the addition law; or (3) a single complementary event are accessible to students in the Piagetian Stage IIIA (early formal operations).

If the assumption is made that the full formal operation stage is reached during and after the age of 16 years, then any probability item involving the multiplication law will cause difficulty if taught to younger students.

3. Research Design and Procedure

Responses from four of the six multiple-choice probability items from the 1973 and 1974 papers of the G.C.E. were analyzed (1 from 1973 and 3 from 1974). Students from the same schools were looked at each year. The schools were coeducational with 971 individuals in 1973 (493 boys and 478 girls) and 925 individuals in 1974 (510 boys, 415 girls). The assumption was made that the students had the same learning opportunities, although the researchers qualified that by noting such sampling

may not indicate comparable mathematical ability. Speculation about the discrepancies in item performance was then given.

4. Findings

Item 1 from the 1973 papers was: "A penny is tossed five times. What is the probability that it will come down "heads" on each of the first three times and "tails" on each of the other two?" The item involves the multiplication law and correct responses were listed for 52.1% of the boys and 34.9% of the girls. Of the incorrect responses, more girls than boys chose the distractor involving the addition law (16.5% to 13.2%).

Item 1 from the 1974 papers was: "Which of the following occur(s) with probability 1/2? (1) A score of 2, 3, or 4 on a normal die; (2) Two heads from two coins tossed together; (3) A "spade" or a "club" when a card is drawn from a pack of playing cards." The correct answer to this item does not depend on the multiplication law, and correct responses were made by 67.3% of the boys and 51.3% of the girls.

The other two items cited showed similar results. Correct responses on item 2 from the 1974 papers were given by 44.3% of the boys and by 23.9% of the girls. This item involved the multiplication law. Item 3 from the 1974 papers listed 41.2% of the boys and 26.3% of the girls with correct responses. This item involved the multiplication law and the concept of "without replacement."

The results of all three items of the 1974 papers showed that 21% of the boys and 7% of the girls had correct responses on all three items. Incorrect responses on all three items were given by 18% of the boys and 33% of the girls.

5. Interpretations

The majority of children have difficulty with probability items that involve the simultaneous occurrence of a number of events and/or a joint event in which component events themselves involve the simultaneous occurrence of a number of independent events.

The frequency with which boys and girls chose particular distractors often varied considerably. Girls chose distractors more frequently than boys that were "less justifiable" than the other possibilities. For example, on one item 10.5% of the girls chose a probability greater than one as the correct answer while only 4.5% of the boys chose this alternative.

Evidence from this study indicates that the average age for students attaining the Piagetian State IIIB (full formal operations) may be later than 15 years and that girls reach this stage later than boys.

Critical Commentary

Several concerns about this piece of research, which was in general well done, are summarized below:

(a) The author's explanation of the sex differences in performance on the probability items seemed to be narrowly directed to the possibility of differences in rates of intellectual development. Alternative explanations for the differences did not seem to be vigorously explored. For instance, could the differences be related to differences in the ability to read the language used to state the test items? Also, the assumption that the same opportunities to learn the probability ideas was afforded the same schools can be questioned and should be explored further.

(b) Some of the hypotheses concerning how students were thinking when they chose particular incorrect answers suggested that the students were operating in the full formal operations stage (IIIB), which contradicts the authors' later statements that these students had not attained this level of intellectual maturity.

The authors in this article demonstrated in an exemplary way a penetrating anaylsis of a small collection of related test items. The results, in our opinion, were too often phrased in a conclusion-oriented manner for the special sample of students considered and the small number of related test items available for analysis. However, whether they are called conclusions or data-based hypotheses, they are interesting and important and worthy of further investigation.

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